Sultan Qaboos University
College of Science
Department of Mathematics and Statistics

MATH2107 - **Calculus I** - SPRING 2011

FINAL EXAM - VERSION A

Time allowed : $2\frac{1}{2}$ hours

INSTRUCTIONS:

- *Write your ID#, name and section number on this page.*
- *Write your ID# on each subsequent sheet.*
- *Show all details of your work for full credit.*
- *Attempt all questions, each question on its separate page.*
- *For Multiple Choice Questions, Circle the correct answer.*
- *Graphing calculators are not allowed in this exam.*
- *The last sheet is for rough work which will not be graded.*
- *Do not separate the sheets of this stapled booklet.*

SECTION :.....

DATE:.....

I.D. NUMBER:.....

NAME:.....

Question No.	Max. Mark	Score
MCQ Q-1	20	
Q-2	6	
Q-3	6	
Q-4	6	
Q-5	8	
Q-6	5	
Q-7	6	
Q-8	8	
Q-9	14	
Q-10	8	
Q-11	8	
Q-12	5	
TOTAL	100	

1. [20 Marks] For the following Multiple Choice questions, Circle the correct answer. No partial credit will be given. Each question carries 2 marks.

- If $\lim_{x\to a} f(x) = 2$ and $\lim_{x\to a} g(x) = -1$, then $\lim_{x\to a} [f(x) - 3g(x) + 1]$ is :

  (A) 6 **(B)** 2 **(C)** -1 **(D)** 0 **(E)** None of these

- The function $f(x) = \ln(2 - e^x)$ has a vertical asymptote at :

  (A) $x = \ln 2$ **(B)** $y = 2$ **(C)** $x = 2$ **(D)** $y = \ln 2$ **(E)** None of these

- If $f(x) = \ln\sqrt{4e^{2x}}$, then $f'(x)$ is equal to :

  (A) $\dfrac{1+\ln 4}{2}$ **(B)** 1 **(C)** $1 + \ln 4$ **(D)** $\dfrac{1}{2}$ **(E)** None of these

- $\lim_{h\to 0} \dfrac{\tan\left(\frac{\pi}{4}+h\right) - \tan\left(\frac{\pi}{4}\right)}{h}$ is equal to :

  (A) $\frac{1}{2}$ **(B)** 2 **(C)** -2 **(D)** 1 **(E)** None of these

- $\lim_{x\to\infty} \dfrac{x^4}{24e^x}$ is equal to :

  (A) ∞ **(B)** $-\infty$ **(C)** 0 **(D)** -2 **(E)** None of these

- $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{1-\sin x}{\cos x}$ is equal to :

  (A) 0 **(B)** $-\infty$ **(C)** 1 **(D)** ∞ **(E)** None of these

- If f is continuous on $[-1,1]$ and $f(-x) = -f(x)$, then $\displaystyle\int_{-1}^{1} f(x)dx$ is :

  (A) 0 **(B)** $-2\displaystyle\int_{0}^{1} f(x)dx$ **(C)** $2\displaystyle\int_{0}^{1} f(x)dx$ **(D)** $\displaystyle\int_{0}^{1} f(x)dx$ **(E)** None of these

- The average value of $f(x) = x + \cos x$ on $[0,\pi]$ is :

  (A) π **(B)** $\dfrac{\pi}{2}$ **(C)** 2π **(D)** 0 **(E)** None of these

- If $\displaystyle\int_{1}^{e} \ln x \, dx = 1$ and $a > 0$, then $\displaystyle\int_{1}^{e} \ln(ax)\, dx$ is :

  (A) a **(B)** $(e-1)\ln a + 1$ **(C)** $\dfrac{1}{a}$ **(D)** $-a$ **(E)** None of these

- $\tanh(\ln x)$ is equal to :

  (A) $\dfrac{x-1}{x+1}$ **(B)** $\dfrac{x^2+1}{x^2-1}$ **(C)** $\dfrac{x^2-1}{x^2+1}$ **(D)** $\dfrac{x+1}{x-1}$ **(E)** None of these

2. [6 Marks] Find the value(s) of the constant a such that the function f is continuous at $x = 0$.

$$f(x) = \begin{cases} \dfrac{e^{\tan^{-1} x} - 1}{x} & \text{if} \quad x < 0 \\ a \ln(x+2) & \text{if} \quad x \geq 0 \end{cases}$$

3. [6 Marks] Use the **Squeeze Theorem** to show that $\lim_{x\to 0} \sqrt{x^3+x^2}\ \sin\left(\frac{\pi}{x}\right) = 0$.

4. [6 Marks] Let $f(x) = \sqrt{3x+1}$. Use the **definition** of the derivative as a limit to prove that

$$f'(x) = \frac{3}{2\sqrt{3x+1}}$$

5. $[4+4=8$ Marks] Find $\dfrac{dy}{dx}$:

(a) $y = \ln\left[\dfrac{(x^3+1)^4 \sin^2 x}{\sqrt[3]{x}}\right]$

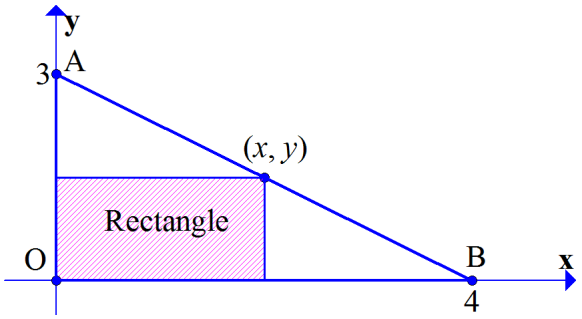
(b) $x^3 + x\tan^{-1} y = e^y$

6. [5 Marks] If $g(x) = [f(x^2+1)]^3$, $f(2) = 2$ and $f'(2) = -3$, find $g'(1)$.

7. [6 Marks] Let $f(x) = \dfrac{x}{x+2}$. Verify that f satisfies the hypotheses of the **Mean Value Thorem** on the interval $[1, 4]$. Then find all numbers c that satisfy the conclusion of the theorem.

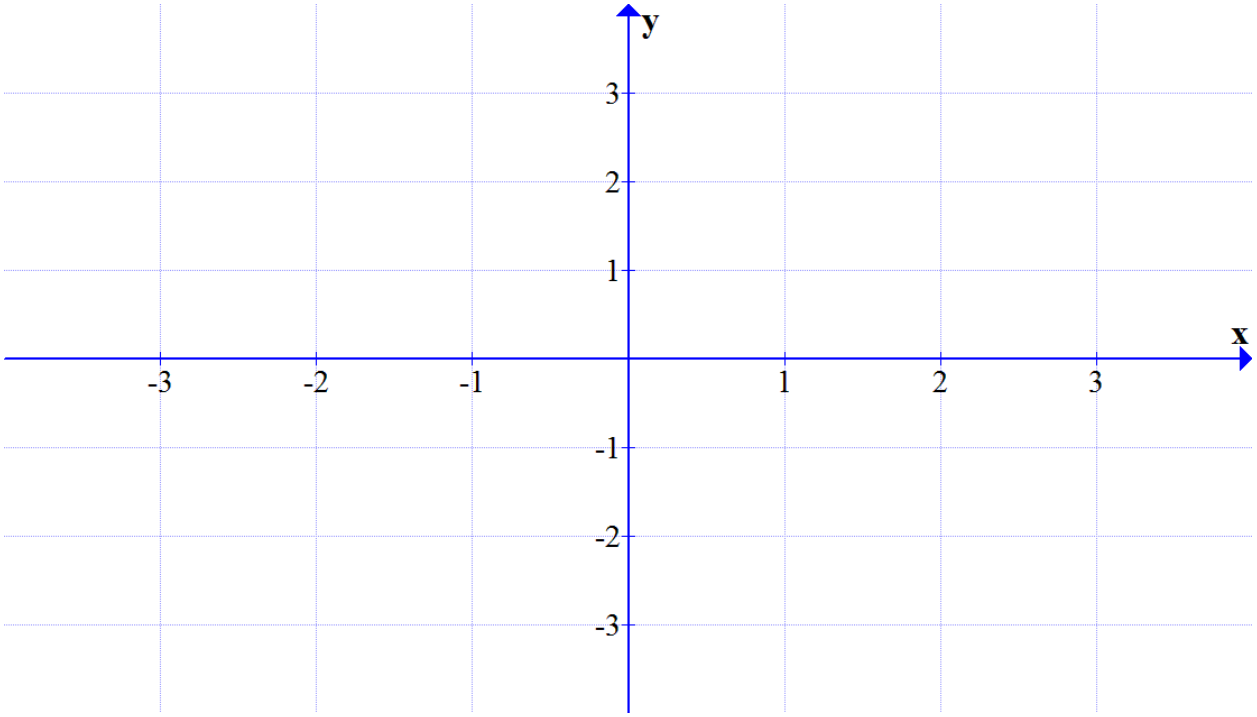
---

8. [8 Marks] In the right triangle AOB, shown below, the side OA is 3 cm long and the side OB is 4 cm long. Find the dimensions of the rectangle of largest area that can be inscribed inside this triangle if two sides of the rectangle lie along these sides of the triangle.
(*Hint*: First find an equation of the line through the points A and B).



9. [14 Marks] Given $f(x) = x^4 - 4x^3 + 4x^2$,
with $f'(x) = 4x\,(x-1)\,(x-2)$ and $f''(x) = 4\,(3x^2 - 6x + 2)$,

sketch the graph of f, showing all details of your work (Asymptotes, End Behavior, Intervals of Increase/Decrease, Local Extrema, Intervals of Concavity, Inflection Points, all Intercepts) and include all the significant features in the graph (Use the next page for the graph).


y
x
3
2
1
-1
-2
-3
-3
-2
-1
1
2
3

10. [8 Marks] Use a **limit of a Riemann sum** to evaluate the exact area between the curve $y = x^2 - 2x + 2$ and the x-axis for $0 \leq x \leq 1$.

11. $[4 + 4 = 8$ Marks$]$ Evaluate the following integrals :

(a) $\displaystyle\int_0^{\frac{1}{2}\ln 3} \frac{e^x}{1+e^{2x}}dx$

(b) $\displaystyle\int_0^{\ln 3} \sinh(x)\cosh^3(x)dx$

12. [5 Marks] If $f(x) = \int_1^x \frac{\cos(t-1)}{t^2+3t+5} dt$, find an equation of the tangent line to the curve $y = f(x)$ at $x = 1$.

Sheet for Rough Work will not be Graded

Sheet for Rough Work will not be Graded